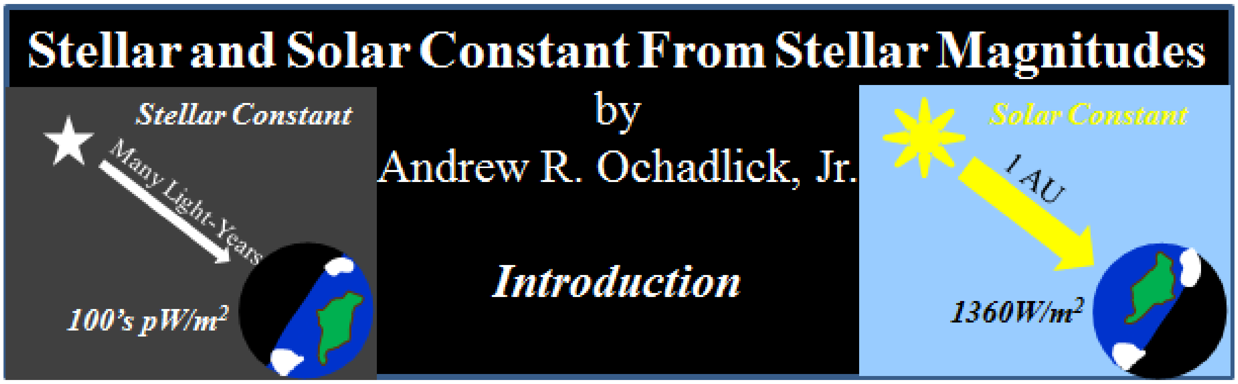


What is meant by "Stellar Constant"? That is made clear by first noting the familiar term of Solar Constant refers to the power per square meter, $\left(\frac{W}{m^2}\right)$, of sunlight on a surface normal to the Sun's direction received at the Earth's distance from the Sun. The term Stellar Constant refers to the power per square meter, $\left(\frac{W}{m^2}\right)$, of star light on a surface normal to the star's direction which we also receive on Earth. Both represent the brightness we see, one represents the blinding brightness of the Sun, the other the comparable faintness of a star. When speaking in the language of "terms and units" regarding brightness, it is easy to get wrapped around the axel, with the terms, and the associated units of luminosity, brightness, intensity, radiance, irradiance, illuminance, radiant flux, etc. Here, the term brightness will be used and can be taken to be equivalent to irradiance.

Using Planck's Law, the solar constant for the Sun at the Earth is easily calculated to be *1,359 W/m*^{2}. The calculation steps are shown in Figure 1 for the Earth-Sun distance of *1 AU,* or 1 Astronomical Unit. Or, equivalently, the brightness of the Sun at 1 AU is: $B_{1AU}^{Sun} = 1{,}359\,\frac{W}{m^2}$, where *1 AU =1.496 x 10*^{11} *m* and *B* represents "brightness".

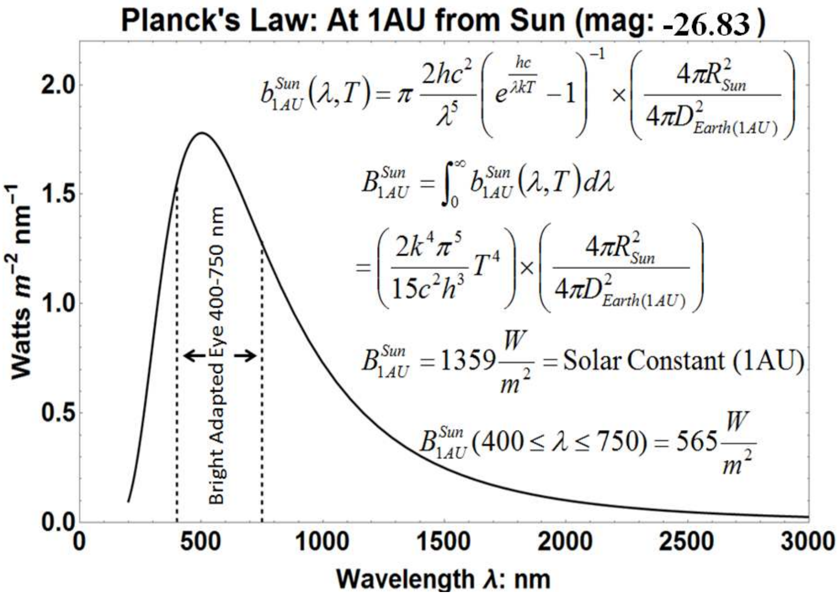


Figure 1: Solar Constant at 1 AU from the Sun

Now suppose we could view the Sun from a distance of 10 pc, or 10 parsec, instead of 1 AU, where 1 pc = 2.062648 x 10^5 AU (of course, we don't need all of those digits). What would the Solar Constant, or equivalently its Stellar Constant, be if the Sun was 10 pc away instead of 1 AU and therefore appear to us as a faint magnitude 4.8 star? Again using Planck's Law, a calculation for 10 pc is shown in Figure 2. (The distance of 10 pc was chosen because that is the distant at which absolute magnitudes corresponds to and the absolute magnitude of the Sun is 4.8.)

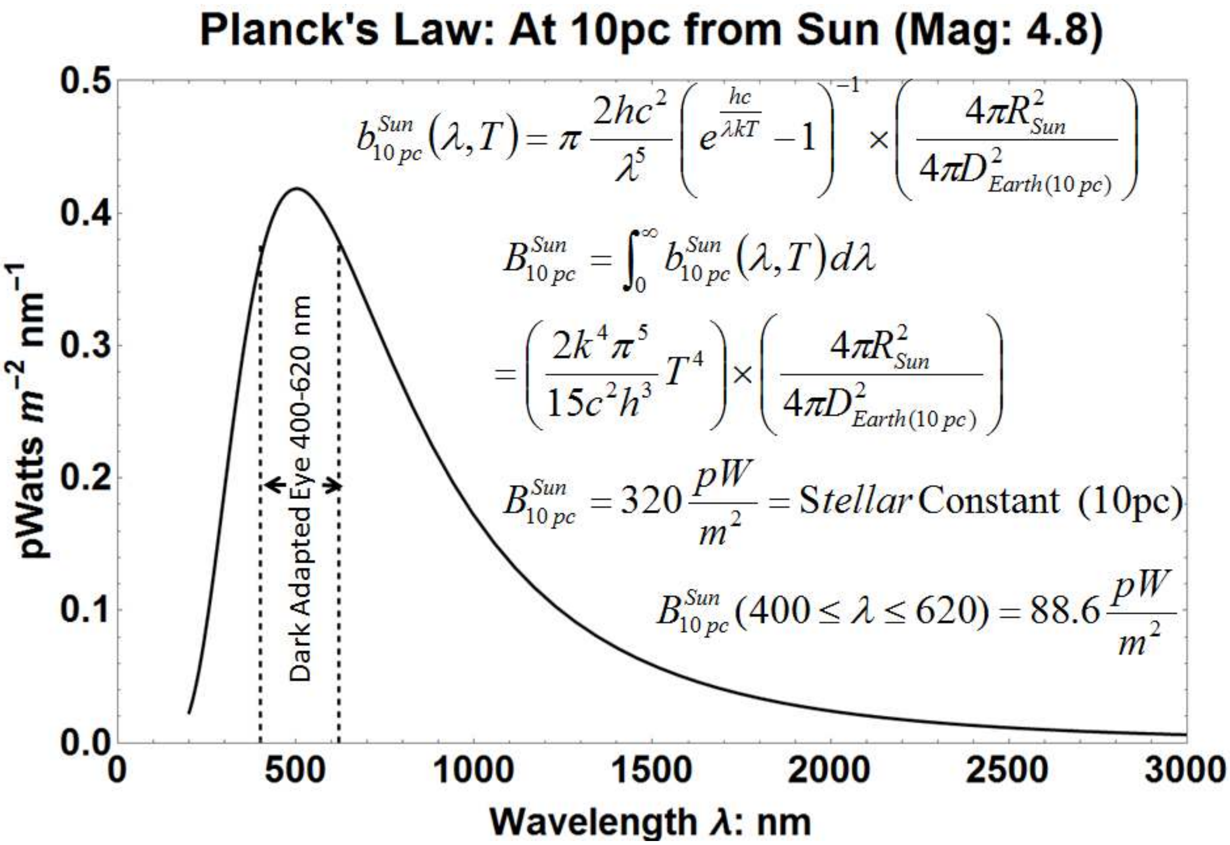


Figure 2: Solar Constant at 10 pc

Using Planck's Law again, but for a much greater distance from the Sun, the solar constant for the Sun at a distance of 10pc can also be calculated as *320 pW/m*^{2}: i.e., $B_{1pc}^{Sun} = 320\,\frac{pW}{m^2}$. *(Both Figures 1 and 2 show an eye sensitive region and the resulting calculation of Plank's law for the eye's "narrow band", such as a range in nm of 400 ≤ λ ≤ 750, instead of the usual zero to infinity for wavelength. The "narrow band" aspects are not disused in this text.)*

Notice: In this text, Planck's Law is used with a factor of π in front to eliminate units of the unitless quantity of "steradians" and therefore the units from Plank's Law in the form used here are W/m^2. Irradiance has the units of W/m^2. My admirable physics professors in the mid-to-late 70's often used "*b*" or "*B*" for brightness when instead of, or in place, of irradiance denoted by "*i*" or "*I*". Obviously, "*i*" causes difficulties because it more often represents $\sqrt{-1}$. I have a fondness for the tradition of "*b*" or "*B*" and the term brightness with units of W/m^2. So if a reader objects to "*b*" or "*B*" and the term brightness with units of W/m^2, they are free to replace my use of "brightness" with "irradiance" or whatever they prefer. Furthermore, on my preference for brightness, if I am showing someone two stars in the eyepiece of my 12 ½ inch, f/8 Newtonian and I say "the one with more irradiance is much further away from us than the one with less irradiance" I'll be asked, "what's irradiance". If I replace "irradiance" with "brightness", experience shows the point is then understood by the visitor as everybody has a "feel" for what brighter means.

Instead of using Planck's Law directly to calculate the "W/m^2" brightness of a star, which is not possible for most stars as neither their temperature nor their diameter nor their distance are known, the objective here is to use stellar magnitudes to determine the "W/m^2" coming to Earth from any star whose magnitude is known. That objective was partially motivated by an intriguing equation in "The Observer's Handbook" published every year by "The Royal Astronomical Society of Canada". Their compact equation is shown here as Equation (1):

$$E(lux) = 10^{-0.4(m+13.99)}. \qquad (1)$$

Associated with their equation is the explanation: "A light source of apparent visual magnitude m provides an illuminance E(lux) where $E = 10^{-0.4(m+13.99)}$." Equation (1) will be developed here, first by discussing magnitudes, then by developing an expression like Equation (1) but in units of "W/m^2". I prefer "W/m^2" over "lux" and so for comparison purposes Equation (1) will lastly be converted from "lux" to "W/m^2".

Stellar Magnitudes and the Equation: $m_2 - m_1 = \frac{5}{2}\ log_{10}\left[\frac{B_1}{B_2}\right]$

For here, and in general, there are two important aspects, or rules, of stellar magnitudes:

i.) brighter stars have a lower numerical value for their magnitude than do fainter stars, and,

ii.) a difference of 5 magnitudes means a factor of 100 in brightness.

So from item i.), a star of magnitude 2 is brighter than a star of magnitude 7,
and from item ii.), the magnitude 2 star is 100 time brighter than the magnitude 7 star.

The magnitude ordering, in what appears to be numerically backward system, can be looked at in terms of a race: the first one over the line is number 1, the second is number 2, the third number 3, and so on. In a similar way the first star you see after sunset is number 1, later you see fainter stars and they are number 2, and when you see number 3 stars it is even darker. So the first few stars you see are roughly magnitude 1, as the sky darkens, the next few star are roughly of magnitude 2, and so on. With the race analogy in mind, perhaps that is why brighter stars have a lower numerical value for their magnitude than do fainter stars?

To develop an equation governing the behavior or relationship of magnitude and brightness, assume there are two stars m_1 and m_2 with respective brightness levels of B_1 and B_2. Next, assume m_1 is the magnitude of the brighter star, then $m_1 < m_2$ via rule i.). Next, by rule ii.) $B_1 > B_2$. In other words, $m_2 - m_1 = 5$ and $\frac{B_1}{B_2} = 100$. Here are the steps to "develop" an equation relating magnitudes and brightness. The symbol $\overset{?}{=}$ is my shorthand indicator for asking what, hence the "?", can be done to make the left and right sides equal, hence the "=".

Step 1: First issue is how to make the number 5 equal the number 100?

$$[m_2 - m_1 = 5] \overset{?}{=} \left[\frac{B_1}{B_2} = 100\right]$$

Step 2: Because the small change of 5 in the difference of magnitudes corresponds to a huge ratio of 100 in brightness, to balance such a "dynamic range" in brightness, consider taking the $\log_{10}$ of the brightness ratio:

$$[m_2 - m_1 = 5] \overset{?}{=} log_{10}\left[\frac{B_1}{B_2} = 100\right]$$

Step 3: That then gives:

$$[m_2 - m_1 = 5] \overset{?}{=} log_{10}\left[\frac{B_1}{B_2} = 100\right] = 2$$

Step 4: To convert the 2 to 5, introduce a factor of $\frac{5}{2}$

$$[m_2 - m_1 = 5] \overset{?}{=} log_{10}\left[\frac{B_1}{B_2} = 100\right]\frac{5}{2} = 5$$

Step 5: Put it all together to obtain Equation (2):

$$m_2 - m_1 = \frac{5}{2}\, log_{10}\left[\frac{B_1}{B_2}\right]. \qquad (2)$$

Step 6: Test Equation (2) with values:

$$m_2 - m_1 = \frac{5}{2}\, log_{10}\left[\frac{B_1}{B_2}\right] \quad\longrightarrow\quad 7 - 2 = \frac{5}{2}\, log_{10}[100] \quad\longrightarrow\quad 5 = 5.$$

The above steps for obtaining Equation (2) are far better than just "pulling the equation out of a hat". *In many books, lectures, or papers, the factor of* $\frac{5}{2}$ *is written as 2.5, which is of course correct. But keeping the integers 2 and 5 in the equation helps to signal and appreciate that a magnitude difference of 5 corresponds to a brightness change of* 10^2*, and keeping the 2 in the denominator can be pointed to as the exponent of 2 in* $10^2 = 100$*. Referring back to Equation (1), in its exponent is the factor 0.4 which is equivalent to* $\frac{2}{5}$*. Again, because of their indisputable significance, my preference is to use the integers 2 and 5 in the equation.*

Stellar Magnitudes in terms of Watts per Square Meter $\left(\frac{W}{m^2}\right)$

By raising the expressions on both sides of Equation (2) to the power of ten, the function log_{10} can be eventually eliminated. To do so begin by taking the factor of $\frac{5}{2}$ to the right side:

$$\frac{2}{5}\,(m_2 - m_1) = log_{10}\left[\frac{B_1}{B_2}\right]. \qquad (3)$$

Next raise both sides to the power of 10:

$$10^{\frac{2}{5}(m_2 - m_1)} = 10^{\,log_{10}\left[\frac{B_1}{B_2}\right]}\ . \qquad (4)$$

Equation (4) simplifies to:

$$10^{\frac{2}{5}(m_2 - m_1)} = \frac{B_1}{B_2}. \qquad (5)$$

Rearranging Equation (5) gives:

$$B_1 = 10^{\frac{2}{5}(m_2 - m_1)} B_2. \qquad (6)$$

Equation (6) has approached the form of Equation (1) and now numerical values need to be selected for inserting into Equation (6). Referring back to the "The Observer's Handbook" published by "The Royal Astronomical Society of Canada", the Solar Constant is listed as

1.360 kW/m² , or equivalently, *1360 W/m²,* which is very close to the *1359 W/m²* in Figure 1, and the magnitude of the Sun is listed as -26.83, bolometric. Since the handbook lists the bolometric magnitude with its "Energy flux" of *1.360 kW/m²*, I have used the two values together here to reproduce a close equivalent to Equation (1).

In Equation (6), let $B_2 = 1360\ W/m^2$ and let $m_2 = -26.83$, then, by using the Sun's properties as a reference, Equation (6) becomes:

$$B_1 = 10^{\frac{2}{5}(-26.83 - m_1)} 1360\ \frac{W}{m^2}. \tag{7}$$

To duplicate the form of Equation (1), the $1360\ \frac{W}{m^2}$ in Equation (7) needs to be moved into the parentheses $(-26.83 - m_1)$ in Equation (7). This can be accomplished in the following way, which is a bit tricky, you can see the term $10^{\frac{2}{5}\frac{5}{2}log_{10}[1360]}$ represents a number and is simply another way of writing the number 1360, so we have:

$$B_1 = 10^{\frac{2}{5}(-26.83 - m_1)} 10^{\frac{2}{5}\frac{5}{2}log_{10}[1360]}\ \frac{W}{m^2}.$$

The factor of $\frac{5}{2}$ seems to be wrong but the other additional factor $\frac{2}{5}$ will cancel it however the additional $\frac{2}{5}$ is factored out first. Thus by adding the exponents the result is (after some well-deserved head scratching):

$$B_1 = 10^{\frac{2}{5}\left(-26.83 - m_1 + \frac{5}{2}log_{10}[1360]\right)}\ \frac{W}{m^2}. \tag{8}$$

The quantity $\frac{5}{2}log_{10}[1360] = 7.83385$ and substituting it into Equation (8) and combining it with *-26.83* gives, then by taking the minus sign to outside the parentheses gives:

$$B_1 = 10^{-\frac{2}{5}(18.996 + m_1)}\ \frac{W}{m^2}. \tag{9}$$

Equation (9) is in the form of Equation (1) except for units. Notice that the units of $\frac{W}{m^2}$ have been carried through, as should be the case for clarity since ten to any power is unitless.

At this point Equation (9) can be re-written in a more general form via a "*" on the magnitude to indicate the fact that a stellar magnitude is involved and the brightness, B_1, can be referred to as the "Stellar Constant" to give:

$$StellarConstant = 10^{-\frac{2}{5}(18.996 + m*)}\ \frac{W}{m^2}\ . \qquad (10)$$

There are a number of different ways Equation (10) could be rewritten and my inclination is to keep the factor $\frac{2}{5}$ because of its connection of the "rules" given earlier. For example, the quantity $-\frac{2}{5}18.996$ can be calculated and placed in as a factor instead as part of an exponent to create the alternative form:

$$StellarConstant = 2.52116 \times 10^{-8} 10^{-\frac{2}{5}m*} \frac{W}{m^2}.$$

And since *nano- watts*, i.e., *nW*, is a reasonable unit when dealing with stars as we see them in the sky, the factor of $2.52116 \times 10^{-8} \frac{W}{m^2}$ can be converted to $25.2116 \frac{nW}{m^2}$.
Doing so would place the equation in the form:

$$StellarConstant = 25.211610^{-\frac{2}{5}m*} \frac{nW}{m^2}$$

As an alternative regarding base-10, instead of using *10* to a power, after a little algebra, Euler's number, e could be used to replace *10* in Equation (10), with further simplification regarding $\frac{2}{5}$, to produce:

$$StellarConstant = 25.208\, e^{-0.92103\, m*}\ nW/m^2.$$

Using the equation $StellarConstant = 25.208\, e^{-0.92103\, m*}\ nW/m^2$, with $m* = 4.8$, give a $StellarConstant = 0.303\ nW/m^2$ or 303 pW/m^2, which can be compared reasonably with the 320 pW/m^2 noted in Figure 2.

Figure 3 shows a comparison, with $25.208\, e^{-0.92103\, m*}\ nW/m^2$ itself, and various approximate forms of $25.208\, e^{-0.92103\, m*}\ nW/m^2$ to show the effects of going deeper into approximations over the magnitude range of *0* to *10*. And Figure 4 is similar but emphasizes fainter magnitudes from *10* to *15*.

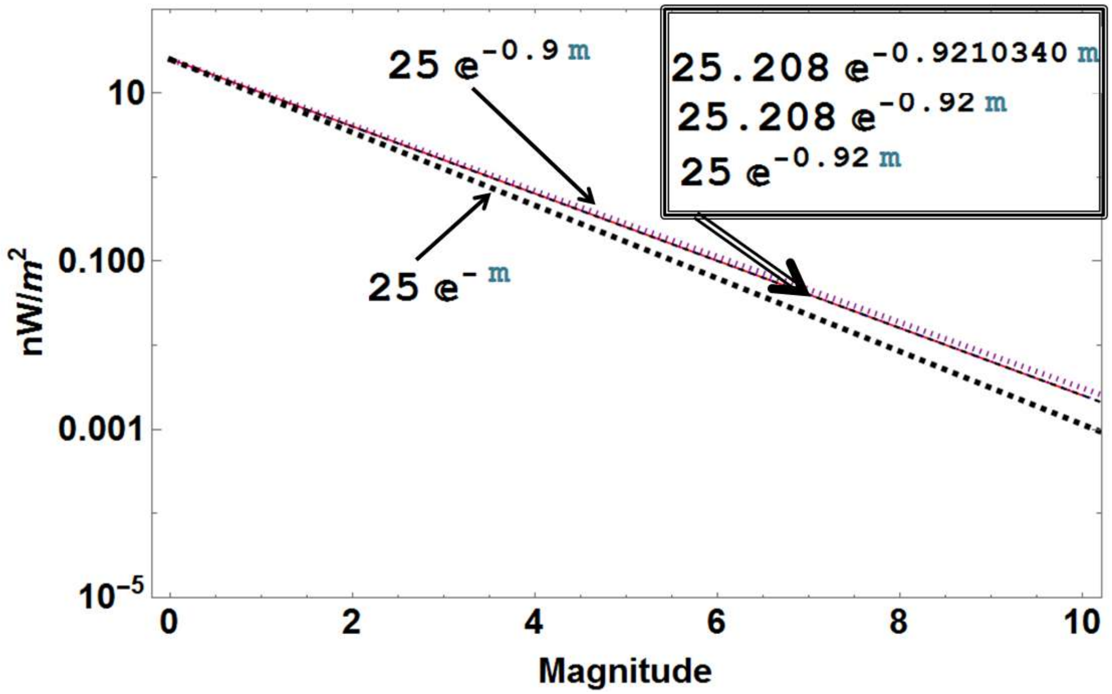


Figure 3: Various Approximations of
StellarConstant = 25.208 $e^{-0.92103\,m^*}$ ***nW/m²*** **Magnitudes 0 to 10**

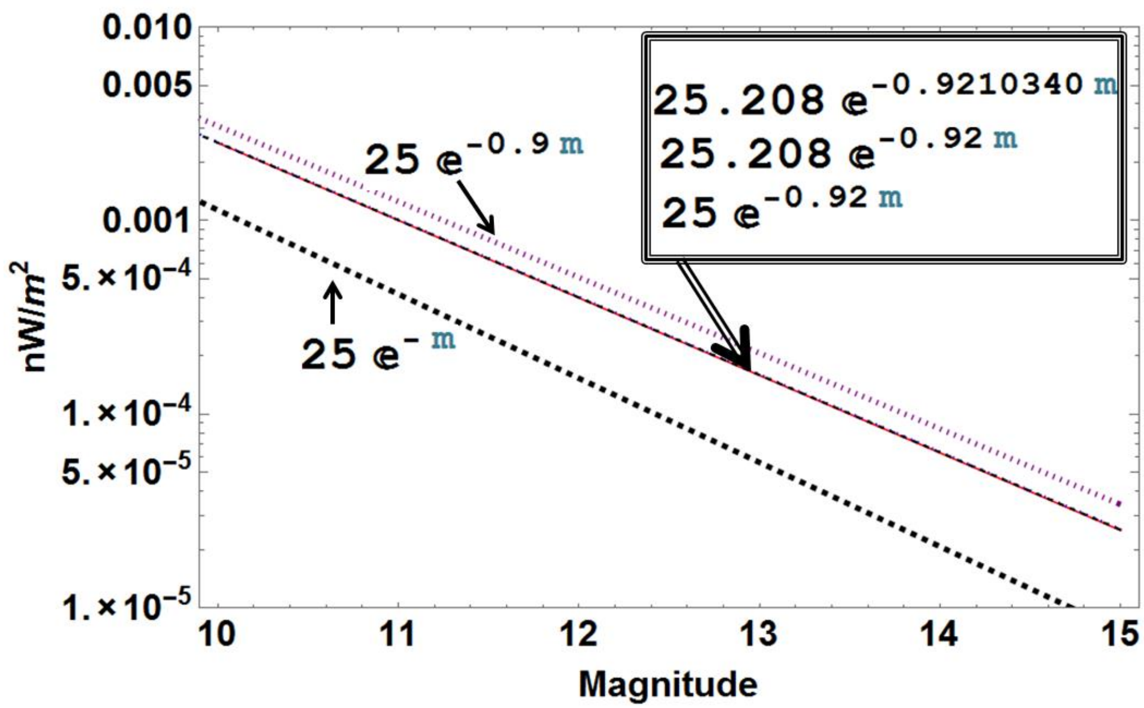


Figure 4: Various Approximations of
StellarConstant = 25.208 $e^{-0.92103\,m^*}$ ***nW/m²*** **Magnitudes 10 to 15**

The most simple form shown Figures 3 to 4 is $StellarConstant \sim 25\, e^{-m^*}\, nW/m^2$ but it is off by about a factor of 5 for faint star of roughly magnitude 15. However, if an order of magnitude is sufficient the simple form $25\, e^{-m^*}\, nW/m^2$ is a reasonable alternative.

Stellar Magnitudes in terms of lux

For converting Equation (10) into units of lux, to be consistent with Equation's (1) form, "The Observer's Handbook" is again useful. The handbook gives the solar constant as 1.36kW/m^2 for a bolometric magnitude of *-26.83* and the solar illuminance as *1.27 x 10*^{5} lux for a visual magnitude of *-26.75*, and in this text, I take *-26.83* as a visual magnitude too. (Besides, to three digits, both are equal to *-26.8*) For the purposes here, despite the slight values in magnitudes for the different set of units, the conversion factor be taken to be "*1.27 x 10*^{5} *lux per 1.36kW/m*^{2}". That simple approach was adopted because discussions on "lux" conversion often conflict with one another.

With the conversion factor selected, Equation (1) can be written in terms of lux as:

$$E(W/m^2) = 10^{-\frac{2}{5}(m+13.99)} lux \left[\frac{1360\, W/m^2}{1.27 \times 10^5\ lux}\right], \quad (11)$$

where the factor of -0.4 appearing in Equation (1) has been re-written as $-\frac{2}{5}$ in Equation (11). Applying a similar trick, with a reversal of what might have been thought as the numerator and denominator, because of a minus sign in the exponent of Equation (11), the conversion factor can be brought into the exponent:

$$-\frac{5}{2} log_{10} \left[\frac{1.27 \times 10^5}{1360}\right] = 4.926. \quad (12)$$

Applying the value from Equation (12) to Equation (11) gives:

$$E(W/m^2) = 10^{-\frac{2}{5}(m+13.99+4.926)} W/m^2, \quad (13)$$

or, after combining the quantiles in the parentheses,

$$E = 10^{-\frac{2}{5}(m+18.92)} W/m^2 \quad (14)$$

which can be compared with Equation (10), repeated here for likeness purposes:

$$StellarConstant = 10^{-\frac{2}{5}(18.996\, +\, m*)}\ \frac{W}{m^2}\ \ . \quad (10)$$

Thus the goal of developing an equation from scratch that is consistent with Equation (1) has been achieved and the equivalent (nearly equivalent) equation is Equation (10).

Example Percentage Difference Between Equations (10) and (14) Using the Magnitude of the Star Vega

The percentage difference between the two exponents, one from Equation (10) and the other from Equation (14), amounts to about 0.4%. But exponents have a big effect as seen in an example for Vega which has a stellar magnitude of 0.0503. Applying Equations (14) and (10), the results for Vega are, respectively:

$$E^{Vega} = 2.58\ \times 10^{-8}\,\frac{W}{m^2}$$ and

$$StellarConstant^{Vega} = \ \ 2.41\ \times 10^{-8} W/m^2\ .$$

The percentage difference between E^{Vega} and $StellarConstant^{Vega}$ is about *6.8%,* where E^{Vega} was adapted from the handbook equation, i.e., Equation (1), and $StellarConstant^{Vega}$, i.e., Equation 10, was developed in this text.

Back to the Sun's Case

For confidence, look at the Sun as a bright star of ***stellar*** visual magnitude -26.83, i.e. $m^* = -26.83$, and plug the Sun's case (at 1 AU) into Equation (10). The result is the Solar Constant from the Sun's stellar magnitude which sort takes us back to an earlier starting point. But it is a good check to confirm Equation (10) is correct:

$$StellarConstant^{Sun} = 10^{-\frac{2}{5}(18.996\, +\, -26.83)}\ \frac{W}{m^2}\ = 1360\,\frac{W}{m^2}\ \ ,$$

as expected.

Comparison with Stellar Data

Table I and II contains Magnitude, Luminosity Relative to Sun, Distance, etc, information on a few stars. The Stellar Brightness (or Irradiance) is in *nW/m*^{2}.

Table I: Stellar Data For Stars with Luminosity Greater or Less Than Sun's

Common Star Name	Visual Magnitude	Luminosity Relative to Sun	Distance Light Years ly	Stellar Brightness at Earth (via Luminosity Rel to Sun)	Stellar Brightness at Earth (via EQ 10)
Aldebaran 13	0.87	350	65	2.82E+01	11.31201352
Alderamin 14	2.45	18	49	2.55E+00	2.639627753
Epsilon Eridani 166	3.72	0.34	10.5	1.05E+00	0.819492367
Epsilon Indi 166	4.69	0.14	11.8	3.42E-01	0.3353873
Kapteyn's Star 272	8.86	0.004	12.78	8.34E-03	0.007203672
Lalande 21185 282	7.49	0.005	8.32	2.46E-02	0.025442016
Menkalinan 325	1.9	95	82	4.81E+00	4.380681932
Menkent 325	2.06	45	61	4.12E+00	3.780436974
Merak 326	2.34	60	79	3.27E+00	2.921073555
Merope 326	4.14	630	389	1.42E+00	0.556603136
Miaplacidus 337	1.67	210	111	5.80E+00	5.414287563
Pollux 400	1.16	32	34	9.43E+00	8.660449309
Tau Ceti 483	3.49	0.62	11.9	1.49E+00	1.012848547
Vega 509	0.03	50	25	2.72E+01	24.52101499

Why have two Tables? The important difference between Table I and Table II is that in Table II all the stars have an extremely higher luminosity than our Sun. Therefore, since our Sun's parameters were used to develop Equation (10), it is expected that Equation (10) may not represent the stars in Table II as well as it does for Table I.

Table II: Stellar Data For Stars with Luminosity Much Greater Than Sun's

Common Star Name	Visual Magnitude	Luminosity Relative to Sun	Distance Light Years ly	Stellar Irradiance at Earth (via Luminosity Rel to Sun)	Stellar Irradiance at Earth (via EQ 10)
Adhara 10	1.5	15000	425	2.83E+01	6.332001662
Alcyone 13	2.85	1400	368	3.52E+00	1.82617918
Alfrik 14	3.23	14600	595	1.40E+01	1.286897785
Bellatrix 59	1.64	2400	243	1.38E+01	5.565975183
Garnet Star 205	4.23	250000	5260	3.08E+00	0.512325304
Menkib 325	3.98	330000	1770	3.59E+01	0.644978694
Nunki 364	2.05	3300	224	2.24E+01	3.815416772
Rho Cassiopeoae 423	4.51	550000	8150	2.82E+00	0.395864263
Rigel 424	0.18	66000	773	3.76E+01	21.35692418
Wezen 520	1.83	50000	1790	5.34E+00	4.672417036
Spica 459	0.98	2100	262	1.04E+01	10.2220996

To compared stellar values in Table I and II to Equation (10), the Brightness (or Irradiance) of each star at the Earth was calculated from the Relative Luminosity of each star (in Column 3), relative to the Sun's. The corresponding distance, *D*, to the star (from Column 4) is used to calculate the spherical area, $4\pi\ D^2$, upon which total power from the star is distributed. The appropriate unit conversion (such as Light Year to meters) are employed to calculate the corresponding $\frac{W}{m^2}$ values from Equation (15) that are found in Column 5, in Tables I and II (but with units of: $\frac{nW}{m^2}$).

$$B = \frac{Sun^{Luminosity}\ Star^{Luminosity}_{Relative\ to\ Sun's}}{4\pi\ D^2}\ . \qquad (15)$$

The Sun's luminosity was taken to be *3.83 x 10*^{26} *W*, so in Equation (15) the quantity $Sun^{Luminosity}$ is $3.83 \times 10^{26} W$. Because of the log scale on Figure 3's vertical axis, Equation (10) is plotted as a straight line in Figure 5 as a function of the visual magnitudes covering the range from 0 to 9. The triangular and square points in Figure 5 representing the values from inserting the values for the stars "Luminosity Relative to Sun", found in Column 5, into Equation (15) as a function of the corresponding stellar magnitudes listed in Column 2 of the Tables. The triangular represent values from Table I and the square points represent values from Table II.

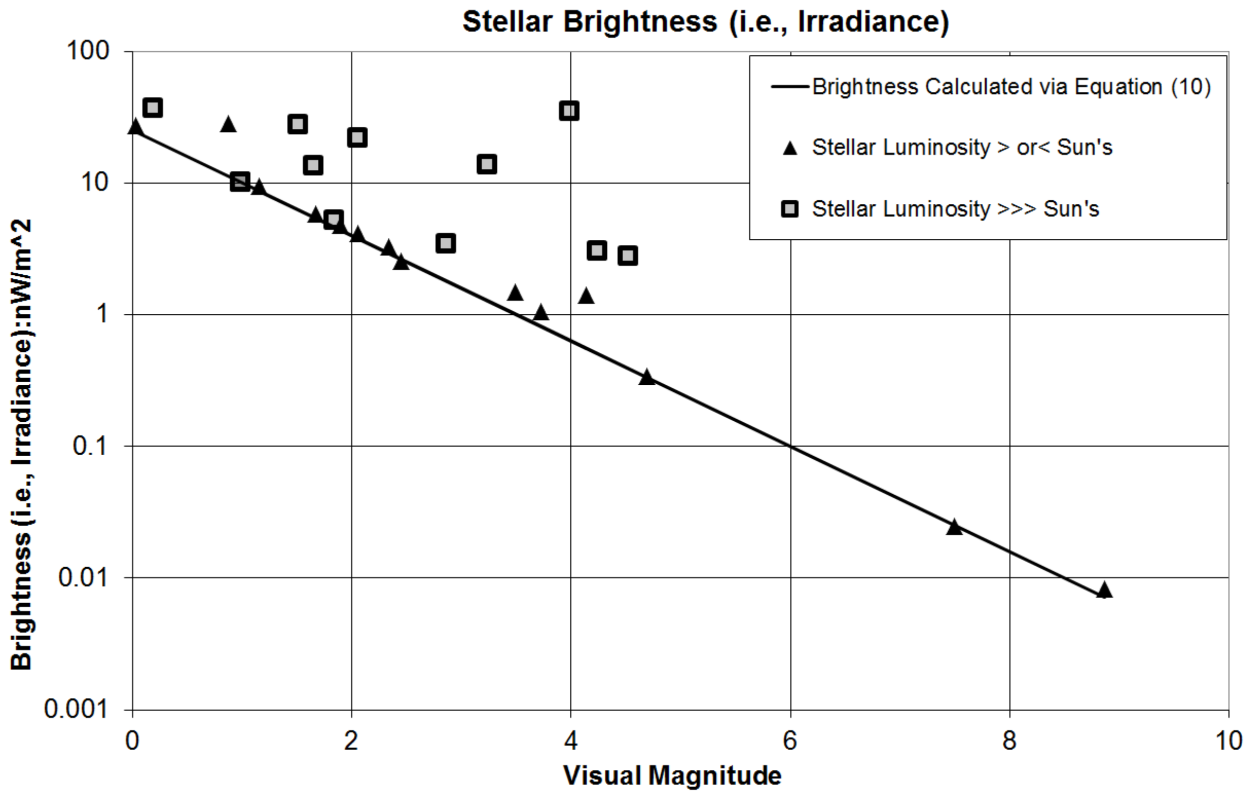


FIGURE 5: Equation (10) is represented by the solid line and compares well with Calculations Based on Stellar Luminosity Relative to the Sun's for Triangular Points but Not as Well for the Square Points

An important aspect of Figure 5 is the two different types of points, triangles and squares, correspond to a grouping of stars in terms of the stars luminosity. Stars (those in Table I) that have a luminosity somewhat greater or less than the Sun are represented by triangles. The other stars (those in Table II) that have a luminosity much-much greater than the Sun are represented by the squares in Figure 5. The triangular points appear to fit the line representing Equation (10) reasonably well for the vertical scale in Figure 5. However, the square points do not. Since the Sun was used as the reference for developing Equation (10), stars with a far-far greater luminosity, and therefore likely different spectral class than the Sun, would not necessarily be expected to conform to Equation (10). However, there are some squares nearly on the line, but in general, the argument that stars with far-far-greater luminosity than that of the Sun are ***not*** likely to fall on the line is reasonably valid.

Another way to display the discrepancy for Stellar Constant values from luminosity and Equation (15) as compared to values from stellar magnitude and Equation (10) is shown in Figure 6. The triangle and squares in Figure 6 represent the same information they displayed in Figure 5.

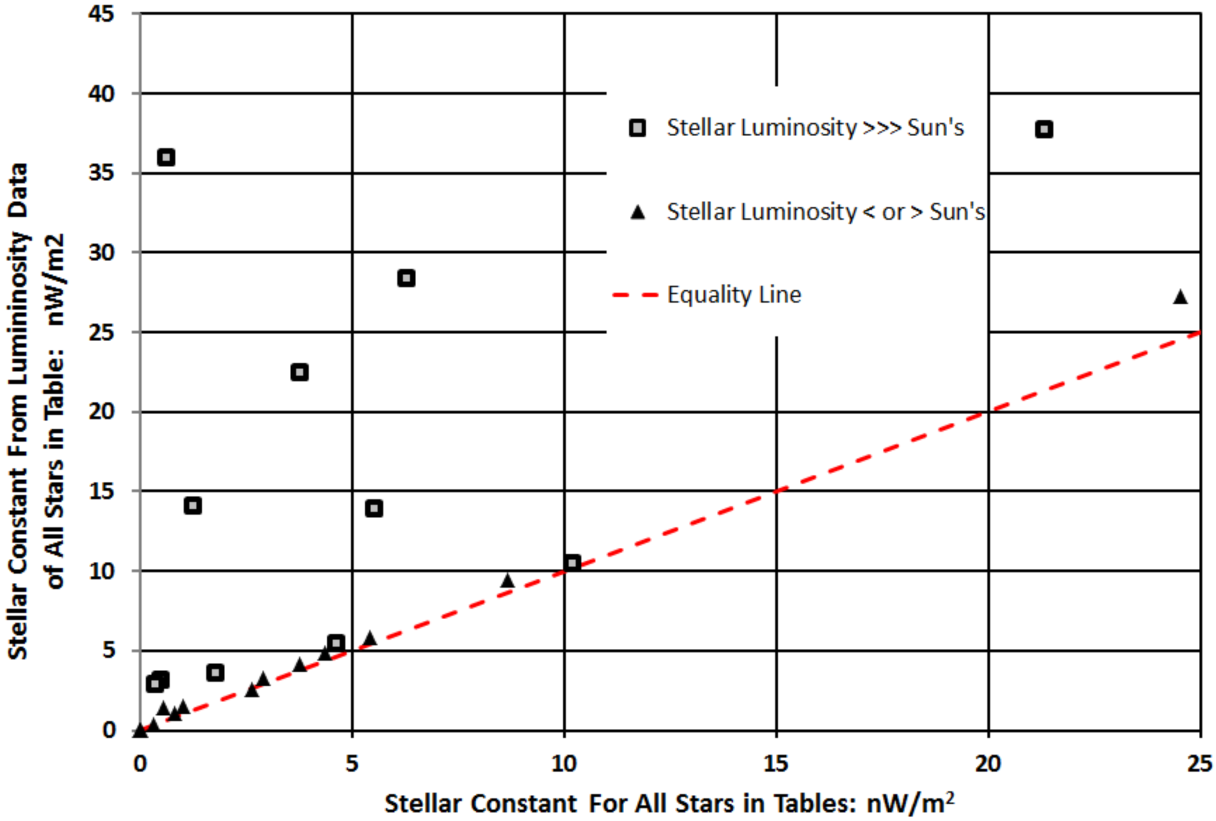


Figure 6: Comparison of Stellar Constant results

In Figure 6, containing the same content as Figure 5, the Stellar Constants calculated from the stellar magnitudes by Equation (10) are represented by the horizontal axis. The Stellar Constants calculated from Equation (15) using the luminosity values in the tables are represented by the vertical axis. The red dashed red line represents an "Equality Line" and it is where all the data points should fall. As seen in Figure 6, the triangular points follow the "Equality Line" far more closely than do the squares. And, as mentioned in regards to Figure 5, that is presumed to be due to the fact that the square points correspond to stars with enormous luminosity as that of our Sun.

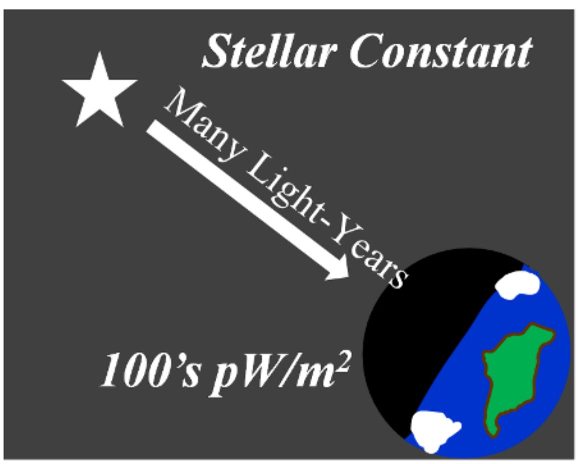
Summary

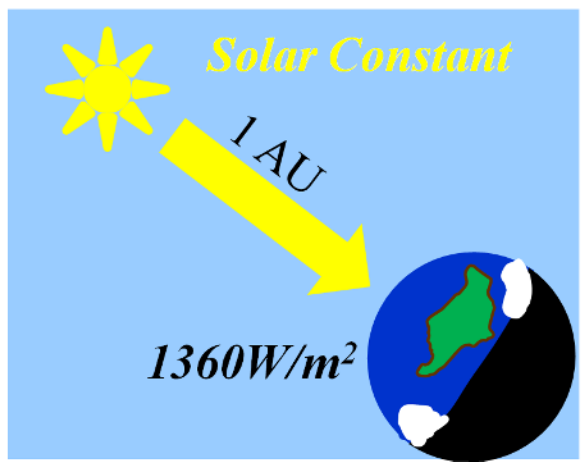
An equation for the stellar constant of a star calculated from the star's magnitude was developed and found to be given by:

$$StellarConstant = 10^{-\frac{2}{5}(18.996 + m^*)}\ \frac{W}{m^2}$$

where m^* represents the star's magnitude.

The Sun's Solar Constant and its magnitude were used to set the numerical parameter in the final equation. The equation was found to be reasonably accurate when tested against stars that had a luminosity comparable to or less than the Sun's and for stars that were not extremely more luminous than our Sun. But "accuracy" was not good for stars with a luminosity extremely greater that of our Sun. The drop in accuracy occurs since our Sun's parameters were used to develop the equation and therefore stars with far more luminosity that also differ significantly from our Sun's spectral class are not accurately by the equation developed here for the stellar constant.





$$\boldsymbol{StellarConstant = 10^{-\frac{2}{5}(18.996 + m^*)}\ \frac{W}{m^2}}$$

OR,

$$\boldsymbol{StellarConstant = 25.208\ e^{-0.92103\,m^*}\ nW/m^2.}$$